Disentangling the Spatiotemporal Structure of Turbulence Using Multi-Spacecraft Data

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1 Synopsis

This white paper prepared for the 2024 Decadal Survey for Solar and Space Physics concerns the importance of research related to multi-spacecraft missions to address fundamental questions concerning plasma turbulence. Turbulence is an inherently multi-scale and multi-process phenomenon, coupling the largest scales of a system to sub-electron scales via a cascade of energy, while simultaneously generating reconnecting current layers, shocks, and a myriad of instabilities and waves. The solar wind is humankind's best resource for studying the naturally occurring turbulent plasmas that permeate the universe, and we have made significant progress characterizing solar wind turbulence. Yet, due to the severe limitations imposed by single-point measurements, we are unable to characterize sufficiently the spatial and temporal properties of turbulence within the solar wind, leaving many fundamental questions about plasma turbulence unanswered. HelioSwarm is a recently selected NASA multi-spacecraft mission concurrently covering a wide range of scales in the solar wind which will allow us to determine directly the spatial and temporal structure of plasma turbulence. However, spacecraft data alone will not provide answers to the fundamental questions facing our community; answering these questions requires dedicated support for research of and related to the novel multi-spacecraft data provided by HelioSwarm. In this white paper, some of these fundamentally important questions that can only be addressed by research related to multipoint *in situ* measurements are presented.

2 Introduction

Turbulence in a magnetized plasma is the primary mechanism responsible for transforming energy at large injection scales into small-scale motions, which eventually dissipate, heating the plasma or accelerating particles. Plasma turbulence is ubiquitous in the universe, and it is responsible for the transport of mass, momentum, and energy in such diverse systems as the solar corona and wind, pulsar magnetospheres, accretion discs surrounding compact objects, the interstellar medium, planet formation, and laboratory fusion devices. Indeed, under one of the four high-level science goals in the 2013 NRC Heliophysics Decadal survey states, to "[d]iscover and characterize fundamental processes that occur both within the heliosphere and throughout the universe," plasma turbulence is identified as a ubiquitous phenomenon involved in the energization of heliospheric and other astrophysical plasmas.

Developing a predictive understanding of plasma turbulence is critical in heliospheric and astrophysical plasmas, both of which are systems in which the microscopic physics of turbulence can significantly impact their macroscopic evolution. For example, long standing questions in heliophysics – such as how the solar corona is heated to temperatures that are orders of magnitude above that of the photosphere and different species contribute to the heating, or how the solar wind is launched from the Sun – remain unanswered after decades of research, because we lack a complete predictive theory of how the energy of turbulent plasma flows and electromagnetic fields is converted into plasma heat, or some other form of particle energization.

The vast majority of the plasma systems in the universe are weakly collisional, necessitating the application of kinetic plasma physics to fully understand them. Yet, kinetic plasma turbulence is an inherently multi-scale and multi-process phenomenon, coupling the largest scales of a system to sub-electron kinetic scales via a cascade of energy and also generating reconnecting current

layers, shocks, and a myriad of instabilities and waves. The broad range of scales and processes encompassed by kinetic plasma turbulence preclude our ability to analytically or numerically model these global systems; therefore, we must turn to laboratory studies of confined plasmas or in situ observations of natural plasmas, such as the solar wind, to advance the field. The solar wind is humankind's best resource for studying the naturally occurring plasmas that permeate the universe, and it is often referred to as a natural laboratory for plasma physics [45]. Since launching our first major scientific spacecraft mission, Explorer 1, in 1958, we have made significant progress characterizing solar-wind turbulence. Yet, due to the severe limitations imposed by single-point measurements, we are unable to characterize sufficiently the spatial and temporal properties of the solar wind, leaving many fundamental questions about plasma turbulence unanswered. Therefore, the time has now come wherein making significant additional progress to determine the dynamical nature of solar-wind turbulence requires multi-spacecraft missions spanning a wide range of scales simultaneously. The recently selected multi-spacecraft NASA mission HelioSwarm will satisfy this requirement by concurrently covering a wide range of scales in the solar wind, thereby providing the necessary in situ data to directly determine the spatial and temporal structure of plasma turbulence. However, data alone will not provide answers to the fundamental questions facing our community; answering these questions requires support for research of and related to the novel multi-spacecraft data provided by HelioSwarm. This research includes not only analysis of HelioSwarm data, but theoretical and numerical simulations in support of the science, as well as laboratory experiments in which multi-point measurements can be made. Only through a coordinated research program guided by in situ data will closure of the fundamental questions facing the turbulence community be possible.

3 Outstanding Questions

3.1 What is the Energy Distribution in Frequency-Wavevector Space?

Measurements of turbulence in frequency-wavevector space provide insight into the dynamics of the plasma. However, all single-point *in situ* measurements rely on Taylor's hypothesis [44], which assumes that the plasma does not evolve in time as it is convected past the spacecraft and establishes a direct connection between the frequency measured in the spacecraft-frame, ω_{sc} , and the wavevector, \mathbf{k} , of the fluctuations. In the near-earth solar wind, the solar wind velocity is mostly radial and typically super-Alfvénic, $v_{sw} \gg v_A$. Thus, observers adopt Taylor's hypothesis by assuming that $|\omega| \ll |\mathbf{k} \cdot \mathbf{v}_{sw}|$, where ω is the plasma-frame frequency, so that the spacecraft-frame frequency fluctuations are interpreted to be related directly to the wavevector of the spatial fluctuations in the plasma frame, $\omega_{sc} \simeq \mathbf{k} \cdot \mathbf{v}_{sw}$ [23, 28]. However, this central assumption has not been well tested in the solar wind, and it indeed fails when the solar wind speed is low compared to the Alfvén speed or when plasma-frame frequencies are high [12, 15], as occurs with whistler or other dispersive fluctuations. The only direct means of evaluating Taylor's hypothesis across a large range of scales is with the analysis of a multipoint measurement in the solar wind to definitively determine if indeed $|\omega| \ll |\mathbf{k} \cdot \mathbf{v}_{sw}|$.

Determining the plasma-frame frequency and wavevector distribution of energy is also fundamental for understanding the dynamics of turbulence. For instance, if on average $\omega(\mathbf{k}) \ll \Omega_{cp}$, dissipation via resonant cyclotron damping is expected to be minimal, where Ω_{cp} is the proton gy-

rofrequency. However, determining the plasma frame frequency requires fully resolving k. Even in cases wherein Taylor's hypothesis is well satisfied, single-spacecraft measurements only provide access to the component of the wavevector along the solar wind flow direction. Existing multi-spacecraft missions such as Cluster have been employed to determine the plasma frame frequency, e.g., [35, 38]; however, these studies have been limited to the approximately fifty, ten-minute intervals of Cluster data that satisfy the necessary conditions to apply multi-spacecraft techniques to the solar wind, namely that the four spacecraft are in a regular tetrahedron [25, 37] and uncontaminated by foreshock particles backstreaming from earth's bowshock. To establish a statistical understanding of the energy distribution in frequency-wavevector space over a broad range of plasma parameters, a dedicated mission whose orbit is chosen to maximize time in the foreshock-free solar wind is necessary. Thus, to unambiguously determine the energy associated with the plasma frame frequency in the solar wind requires analysis of multi-spacecraft data covering a wide range of scales and angles provided by missions like HelioSwarm.

3.2 What Dynamics Drive the Spectral Distribution of Turbulent Energy?

The distribution of energy in wavevector space is a core prediction of most plasma turbulence theories in use today. One popular example of turbulence theories which predict such a distributon is critical balance [10], which assumes that the non-linear decorrelation time is equal to the linear propagation time, $\chi = \tau_L/\tau_{Nl} \sim 1$. This assumption has been modified and extended to include the alignment of velocity and magnetic fluctuations with spatial scale, so called dynamical alignment [2, 3], and most recently intermittency was incororporated in the model of refined critical balance [20, 21]. At the heart of all of these models is the critical balance conjecture. Indeed, some research "suggests that critical balance... is the most robust and reliable of the physical principles underpinning theories of Alfvénic turbulence" [21] but theoretical alternatives to critical balance remain viable. Thus, measuring χ in the solar wind is the first step in testing the validity of all of these critical balance-based turbulence models, but the measurement requires resolving the components of the wavevector both parallel and perpendicular to the *local in scale* magnetic field. Each of the turbulence models also predict different distributions of power in wavevector space, which again requires resolving the full wavevector. Attempts have been made to resolve the wavevector using single-spacecraft measurements, e.g., [11, 19, 22, 29], but these measurements require \mathbf{v}_{sw} and B to be aligned to determine k_{\parallel} , which is a rare occurrence. Therefore, single-spacecraft tests of these models require combining several days to a month of data, while the auto-correlation time in the solar wind is of order one hour or less, likely mixing different plasma and turbulence conditions in the analysis. Cluster has been used to determine the full spectral anisotropy [36]; however, the range of accessible scales were highly limited. Thus, definitively determining the validity of each of these turbulence models or making progress in developing new models requires the analysis of data from a multi-spacecraft mission, where all components of the wavevector can be measured simultaneously, rather than combining many single-point datasets spanning turbulence with widely varying parameters.

3.3 What is the Turbulent Cascade Rate?

The cascade of energy in a plasma can be directly measured using third-order statistics [31, 39], and the cascade rate is related to proton heating in the solar wind [43]. Under a certain set of assumptions, Kolmogorov's third order law is the only exact, non-trivial turbulence result in hydrodynamics [8]. A similar exact result exists for plasma turbulence under a more restrictive set of assumptions. However, the plasma turbulence cascade is anisotropic, and a single spacecraft cannot resolve the anisotropy. Therefore, a multipoint measurement is necessary to properly measure the cascade. A multipoint measurement spanning many scales can not only provide the data necessary to measure the anisotropy, it can permit the calculation of the spatial gradients contained in the third order equation, directly accessing the primitive form of the third order law and for the first time, bypassing any assumptions about isotropy.

3.4 What is the Nature of Intermittency?

Intermittency, or patchiness in space and time, is an essential property of turbulence directly related to the cascade and dissipation of energy. The stationarity assumption made in single-point spacecraft observations necessarily means it is impossible to disentangle the spatial or temporal nature of intermittent fluctuations, and a causal connection between the structures and observed local heating [6, 7, 26, 27, 32] is equally difficult [4]. Single-point measurements also cannot provide information about the 3D structure and nature of the intermittency. Turbulence models like refined critical balance make predictions about the nature and scaling of intermittent structures [20]; however, testing the predictions requires measuring coherence lengths parallel and perpendicular to the local magnetic field. Such a test is not possible with a single spacecraft without combining many epochs of *in situ* data with different plasma conditions. Single-point *in situ* observations have also found that there is a transition "from multifractal intermittent turbulence in the inertial range to non-Gaussian mono-scaling in the dissipation range" [14], which is not a phenomenon observed nor predicted in hydrodynamic or plasma turbulence. A swarm of spacecraft observing the solar wind at a large range of scales could provide the data necessary to directly address open questions regarding the nature and origin of intermittency.

3.5 What is the Spatial Distribution of Turbulent Fluctuations?

Many processes that operate in the solar wind locally generate structures or waves, and turbulence itself is inherently intermittent; however, single-spacecraft measurements cannot disentangle the causality, evolution, or distribution of the processes, because the implicit assumption is that the plasma is stationary over the period of the measurement.

One example of a process that is impossible to characterize fully using single-spacecraft observations is instabilities and wave generation. Wave modes such as the mirror, whistler, and ion cyclotron modes are routinely observed to constitute a small fraction of the solar wind, e.g., [13, 16, 18, 30]. However, most of these modes are not predicted to exist in the solar wind based on traditional models of Alfvénic turbulence, and most are strongly damped under typical solar wind conditions. Many instabilities expected to operate in the solar wind saturate by generating such wave modes, but a causal connection with regions of unstable plasma and the observed modes is difficult with single-spacecraft measurements, although attempts have been made [9].

Also, the sub-dominant modes likely generated by instabilities are difficult to resolve, because they are masked by the more energetic Alfvénic turbulence. These instability-associated modes also provide a non-local means of transferring energy from large scales to small scales, potentially bypassing the turbulent cascade [1, 17, 33]. Despite being sub-dominant energetically, the modes generated by these instabilities are sometimes large-amplitude and can efficiently scatter particles, leading to fluid-like behavior, including viscous dissipation, even in weakly collisional plasmas [5, 17, 34, 46]. The same instabilities can also lead to complete disruption of Alvénic fluctuations [1, 40, 41], which can interrupt the turbulence cascade partially or entirely. Finally, ion cyclotron modes may be an indicator of the recently discovered turbulence helicity barrier [24, 42], which may inhibit the cascade of energy to scales sub-proton scales in the solar wind and result in significant ion heating. Therefore, it is fundamentally important to establish how frequently these modes are present and the causal connection between the modes and progenitor instabilities or the presence of a helicity barrier, which is only possible through the analysis of multipoint observations.

4 Conclusion

In summary, plasma turbulence plays a fundamental role in the transport of energy, mass, and momentum in the universe. Progress in understanding turbulence will benefit many areas, including fusion confinement, interpreting astrophysical observations, space weather, and the coronal heating problem. However, we have reached a point wherein progress is inhibited by the paucity of multipoint *in situ* solar wind measurements available for analysis. **Bringing closure to the spatiotemporal structure of turbulence will be transformative for the field, and it can only be fully addressed with support for the analysis of multipoint measurements provided by missions like the recently funded NASA HelioSwarm, as well as support for the research necessary to interpret the data, including theory, simulation, and laboratory experiments.**

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