



Solar Orbiter Observations of Proton Core and Beam Populations: Evolution, Relative Drifts, and Heating Rates

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Abstract

In situ observations of solar-wind proton velocity distribution functions (VDFs) deviate from thermodynamic equilibrium, exhibiting temperature anisotropies and suprathermal tails. These VDFs are commonly modeled as the sum of two bi-Maxwellian populations, representing a core and a beam, whose relative drift and anisotropies store free energy. With Solar Orbiter's launch in 2020, we have a set of high-resolution measurements of 3D proton VDFs in the inner heliosphere. In this study, we use data from Solar Orbiter's Solar Wind Analyser Proton and Alpha Sensor and the MAG magnetometer to derive a new dataset of core and beam proton bulk properties, using a novel Poisson maximum-likelihood estimation fitting technique. We present the radial evolution of density, bulk velocity, and temperature for slow solar-wind observations from 2023 June to 2025 June, covering heliocentric distances from 0.3 to 1 au. The resulting proton temperature radial profiles deviate from the adiabatic and double adiabatic predictions, and the relative drift between the core and beam populations decreases with radial distance, in agreement with Helios observations. Finally, we quantify the heating rates associated with the reduction of the differential flow between the core and beam populations, Q_{flow} , and evaluate their energetic importance in comparison to the energetics of solar-wind protons in the inner heliosphere. The value of Q_{flow} is comparable to the empirical heating rate required to explain the observed temperature profiles, indicating that the beam deceleration is energetically important.

Unified Astronomy Thesaurus concepts: [Slow solar wind \(1873\)](#); [Heliosphere \(711\)](#); [Plasma physics \(2089\)](#)

1. Introduction

The solar wind is a highly ionized plasma consisting primarily of electrons, protons, and α -particles, with minor contributions from heavier ions. The particle kinetic properties are characterized by their velocity distribution functions (VDFs), which evolve as they propagate away from the Sun. Proton temperature radial profiles from inner heliosphere solar-wind observations have shown that the proton temperature decreases at a lower rate than that predicted by adiabatic or double adiabatic (see G. F. Chew et al. 1956) expansion (E. Marsch et al. 1982b; L. Matteini et al. 2007; P. Hellinger et al. 2013, 2011; D. Stansby et al. 2018; T. Ďurovcová et al. 2019b; J.-B. Dakeyo et al. 2022, 2025; B. A. Maruca et al. 2023). Thus, additional heating mechanisms are required to explain the observed temperature profiles.

In situ observations of the solar wind indicate that proton VDFs deviate from thermodynamic equilibrium, exhibiting anisotropic features and nonthermal components, typically featuring a shoulder or a secondary peak (W. C. Feldman et al. 1973; E. Marsch et al. 1982b). Such distributions are considered to comprise of two different proton components: (1) the core, characterizing the bulk, and (2) the beam, suprathermal protons whose bulk velocity is offset from that of the core population along the direction of the local magnetic field. The temperature anisotropy, $T_{\perp}/T_{\parallel}$, and relative drift between the two proton components, V_{bc} , are sources of stored

free energy in the plasma, and their evolution with distance from the Sun can provide insight into proton heating.

Observations within the inner heliosphere (0.3–1 au) show that, in the fast solar wind ($V_{sw} > 600 \text{ km s}^{-1}$), $T_{\perp}/T_{\parallel}$ decreases with radial distance from the Sun, R , but is constrained at low values by the firehose instability, preventing the anisotropy from growing (E. Marsch et al. 1982b; L. Matteini et al. 2007). In contrast, in the slow solar wind ($V_{sw} < 400 \text{ km s}^{-1}$), $T_{\perp}/T_{\parallel}$ appears uncorrelated with R (E. Marsch et al. 1982b; P. Hellinger et al. 2013). V_{bc} decreases with R in both slow and fast solar wind (E. Marsch et al. 1982b, 1983; P. Hellinger et al. 2013; T. Ďurovcová et al. 2019b). A model of the differential flow between two particle species in the solar wind attributes this deceleration to plasma instabilities and the rotational force (D. Verscharen et al. 2015), and quantifies the associated heating rate, Q_{flow} , due to the release of kinetic energy from this deceleration.

In this study, we use 3D ion VDF measurements from the Solar Wind Analyser (SWA) Proton and Alpha Sensor (PAS; C. J. Owen et al. 2020) on board Solar Orbiter (D. Müller et al. 2020) to derive a new dataset of solar-wind core and beam proton bulk properties. We use this dataset to investigate the radial evolution of the core and beam proton properties in the slow solar wind. Additionally, we assess the importance of the core-to-beam differential flow relative to other contributions to proton energetics.

To derive the core and beam properties, we use a novel fitting technique based on Poisson maximum-likelihood estimation (PMLE) (M. R. Stoneking & D. J. Den Hartog 1997). Unlike traditional least-squares fitting methods commonly used to fit VDFs (B. L. Alterman et al. 2018; D. Stansby et al. 2018; T. Ďurovcová et al. 2019b), PMLE accounts for the Poisson counting statistics governing



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measurements from particle detectors such as SWA-PAS. This is particularly important for deriving the beam properties, which are resolved from measurements with large uncertainty (small number of counts), where least-squares methods can yield biased fit parameters (M. R. Stoneking & D. J. Den Hartog 1997; G. Nicolaou et al. 2024b). We discuss the advantages of PMLE over least-squares fitting in more detail in Section 2.2.

The rest of the paper is structured as follows. In Section 2, we outline the PAS dataset, the data preprocessing routine and describe the PMLE fitting method and its implementation. In Section 3, we present the radial profiles of the core and beam proton properties and the calculation of Q_{flow} . In Section 4, we discuss our results in the context of previous studies and their implications for solar-wind heating. Finally, in Section 5, we summarize our findings and present our conclusions. Additional technical details are provided in the Appendix.

2. Data and VDF Processing

2.1. The Solar Wind Analyser Proton and Alpha Sensor

PAS is a top hat electrostatic analyzer, measuring the number flux of solar-wind ions to determine their VDF. It makes measurements in a 3D grid resolved in 96 energy-per-charge (E/q) bins, ranging from 200 eV/q to 20 keV/q, 9 polar angle bins, Θ , ranging from -22.5° to 22.5° , and 11 azimuthal angle bins, Φ , ranging from -24° to 42° . The number of detected counts, C , for each bin is transformed to VDF, f , via

$$f(E, \Theta, \Phi) = \frac{m_p^2 C(E, \Theta, \Phi)}{2\Delta t E^2 G(E, \Theta, \Phi)}, \quad (1)$$

where E is the kinetic energy of the protons sampled in each specific instrument energy bin, Δt is the acquisition time, m_p is the proton mass, and G is the effective geometric factor. VDF measurements are made over a period of 1 s, typically at a cadence of 4 s but occasionally at a cadence of 1 s.

2.2. Poisson Maximum-likelihood Estimator

We use a similar approach to G. Nicolaou et al. (2024b) to fit the PAS VDF data. We model each pixel measurement as a Poisson random variable, meaning that the probability of observing C counts in a pixel, $P(C; \lambda)$, is

$$P(C; \lambda) = \frac{\lambda^C e^{-\lambda}}{C!}, \quad (2)$$

where λ is the expected number of detected particles and is determined by the underlying VDF. Thus, the likelihood of observing a given configuration of counts, C_i , across all the detector pixels, i , corresponding to the full 3D measurement with expected counts, λ_i , is given by

$$\mathcal{L}(C_i; \lambda_i) = \prod_i P(C_i; \lambda_i) = \prod_i \frac{\lambda_i^{C_i} e^{-\lambda_i}}{C_i!}. \quad (3)$$

We consider the VDF in a reference frame aligned with the local magnetic field, while remaining in the inertial spacecraft frame (SRF). Thus, we model λ_i using a double bi-Maxwellian

proton VDF, $f_M(u_{\perp,1}, u_{\perp,2}, u_{\parallel}) = f_c + f_b$, where

$$f_c = \frac{n_c}{\pi^{3/2} v_{th,\perp,c}^2 v_{th,\parallel,c}} \exp\left(-\frac{(u_{\perp,1} - V_{\perp,1})^2}{v_{th,\perp,c}^2} - \frac{(u_{\perp,2} - V_{\perp,2})^2}{v_{th,\perp,c}^2} - \frac{(u_{\parallel} - V_{\parallel,c})^2}{v_{th,\parallel,c}^2}\right), \quad (4)$$

and

$$f_b = \frac{n_b}{\pi^{3/2} v_{th,\perp,b}^2 v_{th,\parallel,b}} \exp\left(-\frac{(u_{\perp,1} - V_{\perp,1})^2}{v_{th,\perp,b}^2} - \frac{(u_{\perp,2} - V_{\perp,2})^2}{v_{th,\perp,b}^2} - \frac{(u_{\parallel} - V_{\parallel,b})^2}{v_{th,\parallel,b}^2}\right), \quad (5)$$

where n is the number density, V is the bulk velocity, and $v_{th} = \sqrt{2k_B T_p / m_p}$ is the thermal speed where k_B is the Boltzmann constant, T_p is the proton temperature, and m_p is the proton mass. The subscripts c and b refer to the core and beam populations, respectively, while the subscripts $\perp$ and $\parallel$ refer to directions perpendicular and parallel to the local magnetic field. The velocity components $u_{\perp,1}$, $u_{\perp,2}$, and $u_{\parallel}$ are the three components of the velocity vector defined in the local magnetic field frame. In this model, we assume the same perpendicular bulk velocities for both populations (i.e., no perpendicular drift between core and beam), which is why $V_{\perp,1}$ and $V_{\perp,2}$ have neither the c nor the b subscript. Thus, the model has a total of 10 free parameters: n_c , $V_{\perp,1}$, $V_{\perp,2}$, $V_{\parallel,c}$, $v_{th,\perp,c}$, $v_{th,\parallel,c}$, n_b , $V_{\parallel,b}$, $v_{th,\perp,b}$, and $v_{th,\parallel,b}$, which we collectively denote by ξ . We then calculate λ_i by transforming f_M into counts using Equation (1).

To determine the optimized set of parameters, ξ_{opt} , that best fit the observations, we must maximize the likelihood in Equation (3). It is more numerically convenient to minimize the negative log likelihood expressed as

$$-\ln \mathcal{L}(C_i; \lambda_i(\xi)) = \sum_i (\lambda_i(\xi) - C_i \ln \lambda_i(\xi) + \ln C_i!), \quad (6)$$

since the natural logarithm is a monotonically increasing function.

Equation (6) defines the cost function used in our PMLE fitting routine, and incorporates the Poisson statistics of the data. In contrast, the commonly used least-squares cost function assumes normally distributed errors, which is inappropriate for Poisson-distributed data with low counts. This is demonstrated by G. Nicolaou et al. (2024b), who fit synthetic data based on typical solar-wind parameters using two least-squares methods across different count regimes. Both methods perform poorly at low counts, either underestimating or overestimating the true parameters. By comparison, applying PMLE to the same data accurately recovers the plasma parameters across all count regimes. Furthermore, chi-squared fitting techniques may falsely create a correlation between the fit parameters (G. Nicolaou et al. 2024a). Thus, PMLE is a more suitable method for fitting particle VDFs, especially when deriving proton beam properties, which are characterized by low count statistics. A similar method was recently used by R. Ramstad et al. (2026) to fit 3D ion VDF measurements from MAVEN's SWIA instrument.

2.3. Data Preprocessing

We use the PAS level 2 VDF and level 1 count data from 2023 June 1 until 2025 May 31. Solar Orbiter performed four orbits around the Sun during this period, ranging from 0.3 to 1 au, with a total of $\approx 3.4 \times 10^7$ PAS measurements. For the 3D magnetic field data, we use the level 2 observations of the magnetic field vector from the normal mode of the Solar Orbiter magnetometer (MAG) (T. S. Horbury et al. 2020). Observations are made at a cadence of eight vectors per second, which we average over the PAS measuring time of each observation, matching them in resolution.

Using the magnetic field data, we rotate the velocity grid from the spacecraft frame into the local magnetic field frame. The effective geometric factor for each pixel, G_i , is required to transform the model from VDF to counts. We calculate G_i using Equation (1) and the VDF and count data with $\Delta t \approx 1/(96 \times 9)$ s, the measuring time of each PAS pixel (A. Fedorov 2020). The G_i for data with zero counts is also required to perform Poisson fitting, but cannot be calculated empirically using Equation (1), since $C_i = 0$. To overcome this issue, we set G_i to be the average geometric factor over a 1 day interval centered on the corresponding measurement, as G_i does not vary significantly over such a time interval.

PAS also measures α -particles in the solar wind, which we do not account for in our proton VDF model. An α -particle traveling at the same velocity as a proton carries double the proton energy per charge and thus appears as a secondary peak in the energy spectrum. We employ the K-means clustering technique (S. Lloyd 1982; D. Arthur & S. Vassilvitskii 2007) to remove data likely associated with the α -particles from our fitting routine, as outlined in Appendix A.1. A similar method for separating the energy spectra of electron populations was introduced by M. R. Bakrania et al. (2020) for Cluster PEACE data and used by J. B. Abraham et al. (2022) for Parker Solar Probe SPAN data and X. Wu et al. (2026) for Solar Orbiter SWA-EAS data.

As PAS does not distinguish between different ion species, we cannot guarantee that all α -particle contamination is removed by this method. However, the α -particle population is generally well separated from the proton population in energy space, and thus, we expect α -particle contamination to be minimal. To assess whether α -particles may significantly influence our results, we compare our fitted total proton bulk properties, over a short time interval, with proton parameters obtained using the Gaussian mixture model (GMM) technique developed by H. Ran et al. (2026), which explicitly accounts for the presence of α -particles. We find good agreement between the two methods, suggesting that our method is not significantly influenced by α -particle contamination.

2.4. Fitting Procedure

We fit the double bi-Maxwellian model to the PAS 3D VDF data in the local magnetic field frame, using a three-step fitting procedure to determine the optimized set of plasma parameters, ξ_{opt} . First, we perform a fit of only the core population on data where $|\mathbf{u}|$ is lower than the PAS level 2 ground moment bulk velocity, $|\mathbf{V}_{\text{mom}}|$. This provides a first estimate of the core parameters on data where the beam contribution is minimal. We use the PAS level 2 ground moment density, bulk velocity, and temperature as an indication of initial guesses and bounds for the core fitting parameters.

Second, to estimate the beam parameters, we fit the full model (core + beam) on all the data using the core parameters determined from step 1 as initial guesses, and only allow for the beam parameters to vary. As an initial guess, we set $n_b/n_c = 0.15$ and $V_{b,\parallel} = V_{c,\parallel} \pm V_A$, where $V_A = |\mathbf{B}|/\sqrt{\mu_0 n_p m_p}$ is the local Alfvén speed (proton contribution only) determined using the local magnetic field and the level 2 ground moment density data, and μ_0 is the permeability of free space. The $\pm$ sign in the initial guess for $V_{b,\parallel}$ is determined by the sign of $V_{c,\parallel}$ such that $|V_{b,\parallel}| > |V_{c,\parallel}|$. We determine the initial guesses for n_b/n_c and $V_{b,\parallel}$ by testing a range of values on a small subset of the data, based on the range of values reported for the slow solar wind by P. Hellinger et al. (2013).

Finally, to further refine the core parameters, we perform a fit on all the data points, using the core and beam parameters determined from steps 1 and 2 as initial guesses, only allowing for the core parameters to vary. The core and beam bulk velocities are then transformed into the radial, tangential, normal (RTN) frame, adjusting for the spacecraft velocity. In all fitting steps, we use the *Powell* local minimization algorithm (M. J. D. Powell 1964) implemented in the *scipy.optimize.minimize* Python library (P. Virtanen et al. 2020).

2.5. Fit Evaluation and Filtering

To assess the goodness of fit, we determine the Poisson deviance, D (H. Heinzl & M. Mittlböck 2003; P. McCullagh 2019), which is a measure of the distance between the likelihood of the fitted model and the likelihood of the saturated model, which consists of as many free parameters as data points (i.e., $\lambda_i = C_i$ for all i). Appendix A.2 details how we calculate D and use it to filter out poor fits from our analysis.

We only include fits in our analysis where a prominent beam population is found. We assert that $n_b/n_c > 0.01$, and $V_{bc} > 0.05|\mathbf{V}_{\text{mom}}|$. Additionally, we define an overlap parameter, O , to quantify the overlap between the core and beam populations in velocity space. Fits with a high degree of overlap are excluded from our analysis because they result in unreliable beam parameters, since the beam population is not well resolved from the core population. Appendix A.3 details how we calculate O and apply it to filter our data.

Moreover, we exclude from further analysis measurements where the total proton bulk speed, $|\mathbf{V}_p|$, is below 300 km s^{-1} in the SRF, as PAS measurements are less reliable in this regime (A. Fedorov 2020; G. Lewis et al. 2023). We focus on the slow solar-wind regime, only including fits where $|\mathbf{V}_p| < 500 \text{ km s}^{-1}$ in the RTN frame. After applying all the filtering criteria, we are left with a total of $\approx 1 \times 10^7$ good fits with a prominent beam population for our analysis, corresponding to 47% of the VDFs in the slow solar-wind regime.

Figure 1 shows an example fit of the double bi-Maxwellian model to a PAS 3D VDF measurement, made on 2023 June 2, 12:47:10.470763 UT. For visualization and direct comparison with the data, the fit and data VDFs are collapsed into 1D by integrating over the Θ and Φ angle bins. Appendix A.4 details how we determine the total proton bulk density, velocity, and temperature (zeroth-, first-, and second-order moments), from the subpopulation bulk properties determined by the fits (which are the respective zeroth-, first-, and second-order moments of each subpopulation). We cross-check these fitted bulk properties against the corresponding moments for the entire filtered dataset, as detailed in Appendix A.5.

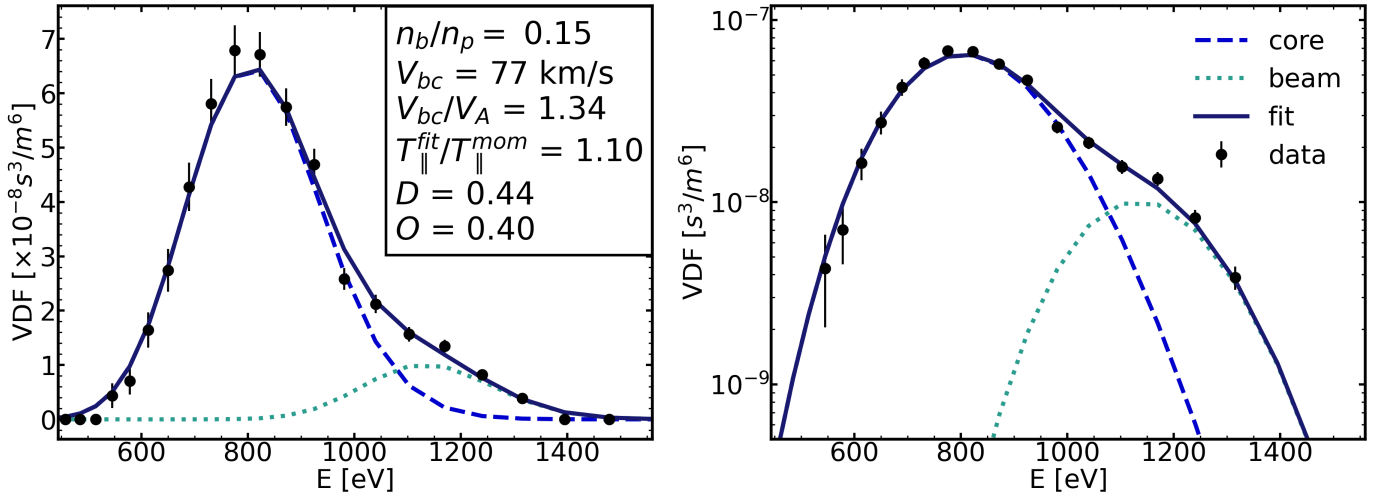


Figure 1. The fit of the double bi-Maxwellian model to the proton only part of a PAS VDF measurement made on 2023 June 2, 12:47:10.470763 UT in linear (left) and logarithmic (right) scales. The data and fit VDFs are collapsed into 1D by integrating over the Θ and Φ angle bins. The uncertainties in the VDF data are estimated assuming Poisson statistics and are given by $f/\sqrt{C}$. The beam abundance, n_b/n_p , core-to-beam drift, V_{bc} , and its normalization to the Alfvén speed, $V_A = B/\sqrt{\mu_0 m_p n_p}$, parallel temperature ratio between the fit and the PAS moments, $T_{\parallel}^{\text{fit}}/T_{\parallel}^{\text{mom}}$, reduced deviance, D , and overlap metric, O , of the fit are also shown.

3. Results

3.1. Radial Profiles

To characterize the radial profiles of the parameters of each proton population, we perform a simple linear regression in logarithmic space, such that a parameter, X , is dependent on R via

$$\log_{10} X = m \log_{10} R + I, \quad (7)$$

where m and I are parameters determined by the fit. The fit is performed on all the data points without any binning. Given the large number of data points, the resulting uncertainties in m and I are negligible (4 orders of magnitude smaller than the fit parameters themselves); thus, we do not report them in the text.

Figures 2–6 show 2D histograms of n , $|V|$, T , $T_{\parallel}$, and $T_{\perp}$, respectively, against R , for the core (c), beam (b), and total protons (p). The histograms consist of 25 linearly spaced bins in R , where vertical columns are normalized to their maximum value to account for the irregular sampling size across the different R bins. The mean and standard deviation for each vertical column, the line of best fit, and the value of m are also shown in the plot. Table 1 summarizes the fitted radial profiles for each parameter and proton population.

Figure 2(d) shows the radial profile of the beam abundance, n_b/n_p . Figure 3 shows the radial profile of the core-to-beam drift speed, $V_{bc} = |V_{\parallel,b} - V_{\parallel,c}|$ in panel (c), which we find to be

$$V_{bc} = 72 \text{ km s}^{-1} \left(\frac{R}{1 \text{ au}} \right)^{-0.56}, \quad (8)$$

along with the radial profile of V_{bc} normalized to V_A in panel (d). Figure 4(d) shows the radial profile of the core-to-beam temperature ratio, T_b/T_c . Figure 5(d) shows the fractional contributions to the second-order moment, $T_{\parallel,p}$, of the total proton population, arising from terms associated with the core and beam parallel temperatures, $T'_{\parallel,c}$ and $T'_{\parallel,b}$, and the core and beam parallel drifts, $V'_{\parallel,c}$ and $V'_{\parallel,b}$ (see Appendix A.6 for

details). Finally, Figure 6(d) shows the radial profile of the temperature anisotropy, $T_{\perp,p}/T_{\parallel,p}$.

The average n_c and n_b radial profiles are well approximated by spherically symmetric expansion with constant radial velocity following a profile of $1/R^2$, with the beam having a slightly steeper profile, similarly to Helios observations (T. Ďurovcová et al. 2019b). The beam abundance, n_b/n_p , is approximately 10% across all R , also in agreement with previous studies of Helios observations (E. Marsch et al. 1982b; P. Hellinger et al. 2013; T. Ďurovcová et al. 2019b). While the n_b profile is slightly steeper than the n_c profile, no clear radial trend is observed in n_b/n_p , possibly due to the large spread in n_b/n_p at each R bin.

The average bulk speed profiles in Figure 3 show that $|V|_c$ increases marginally during expansion, while the $|V|_b$ decreases, matching Helios slow wind observations (T. Ďurovcová et al. 2019b). A decrease of V_{bc} with R is also observed, in agreement with the core and beam bulk speed profiles, also observed in the Helios observations. The drift normalized to the local Alfvén speed increases with radial distance, from an average of 1 at 0.3 au to 1.5 at 1 au, which is in good agreement with the range of values of V_{bc}/V_A observed from Helios (C.-Y. Tu et al. 2004; T. Ďurovcová et al. 2019b).

The average temperature of the beam decreases faster than the core, as seen in Figures 4–6, with the difference being more pronounced in $T_{\parallel}$. The beam is on average hotter than the core, and T_b/T_c is larger at smaller radial distances and decreasing with R , stabilizing at ≈ 1.5 beyond 0.55 au. $T_{\parallel,b}$ decreases at a higher rate compared to $T_{\perp,b}$, while the opposite trend is observed for $T_{\parallel,c}$ and $T_{\perp,c}$.

Qualitatively, the parallel temperature profiles are in agreement with Helios observations of the slow solar wind (T. Ďurovcová et al. 2019b), where the $T_{\parallel,b}$ profile is significantly steeper than the $T_{\parallel,c}$ profile. However, the deviation in the slope between these profiles is more pronounced in our results, where they report $T_{\parallel,c} \propto R^{-0.67}$ and $T_{\parallel,b} \propto R^{-1.03}$, whereas we obtain $T_{\parallel,c} \propto R^{-0.29}$ and $T_{\parallel,b} \propto R^{-1.31}$. From the fractional contributions to $T_{\parallel,p}$ in Figure 5(d), we find that the beam drift and core parallel

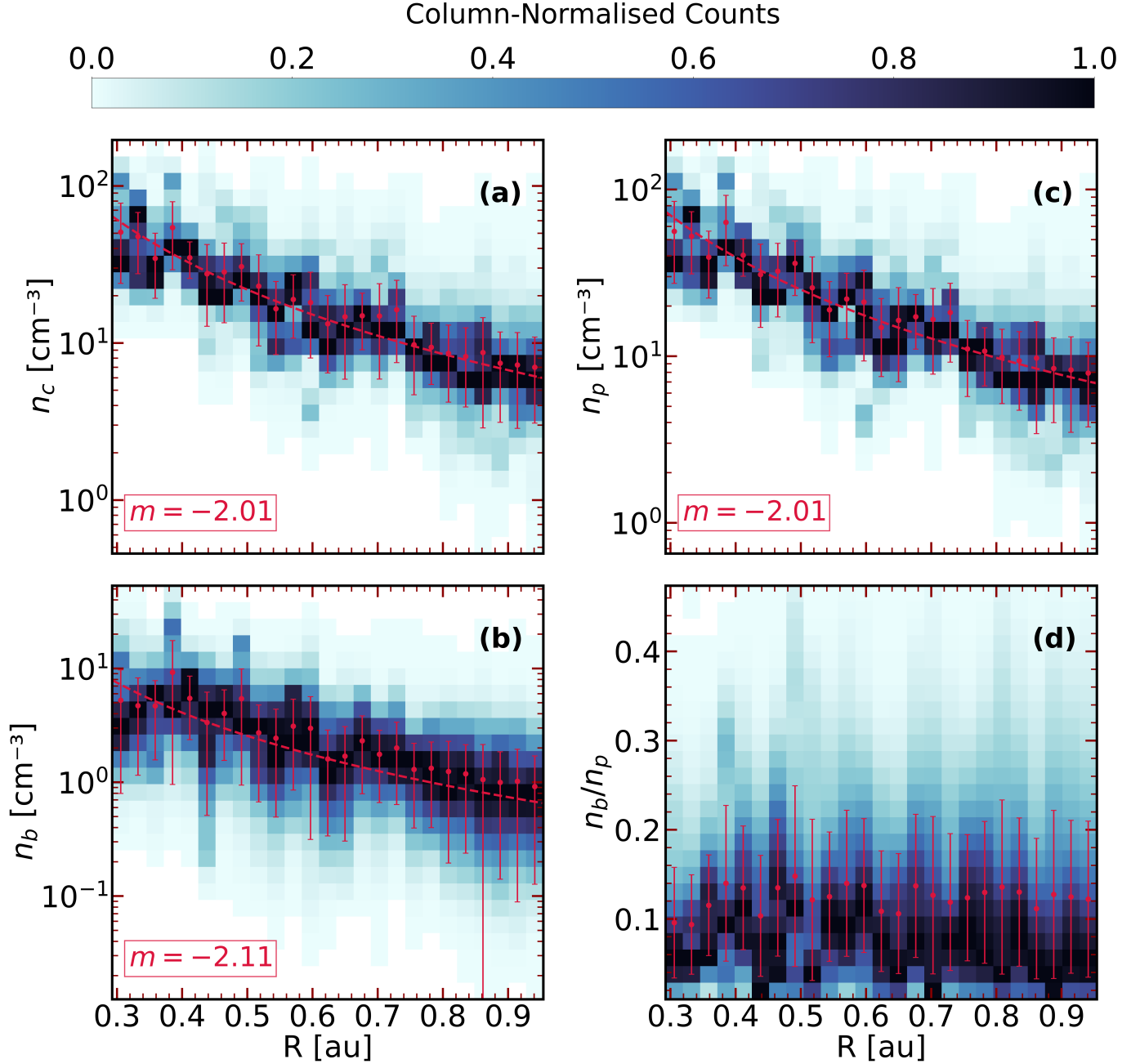


Figure 2. Radial density profiles for: (a) core, (b) beam, (c) total protons. Panel (d) shows the radial profile of the beam abundance, n_b/n_p . Vertical columns are normalized to their maximum value to account for the irregular sampling size across R bins. The red-dashed line in panels (a)–(c) shows the line of best fit using Equation (7), the dots and error bars indicate the mean and standard deviation for each vertical column, and the exponent, m , of the fit is indicated in red text. The fit parameters for the core, beam, and total proton profiles are summarized in Table 1.

temperature contributions are comparable and significantly larger than the beam parallel temperature and core drift contributions. The perpendicular temperature profiles are very similar between the two populations, with the beam profile being slightly steeper than the core profile, which is in contrast to Helios observations where the $T_{\perp,b}$ profile is steeper than the $T_{\perp,c}$ profile (T. Ďurovcová et al. 2019b).

$T_{\parallel,p}$ and $T_{\perp,p}$ decrease at comparable rates, consistent with Helios observations of the slow solar wind (P. Hellinger et al. 2013), although our observations exhibit steeper radial profiles. While $T_{\perp,p}/T_{\parallel,p}$ is larger closer to the Sun, it remains relatively constant at ≈ 0.65 beyond $R = 0.4$ au, reflecting the slow solar-wind Helios observations (E. Marsch et al. 1982b;

their Figure 15). Similarly to the parallel and perpendicular profiles, the T_p profile is steeper than the Helios observations, but shallower when compared to multispacecraft radial trends of the inner heliosphere solar wind (B. A. Maruca et al. 2023).

3.2. Heating Rates

To quantify the rate of energy release due to the decrease of the core-to-beam drift, we adopt the model from D. Verscharen et al. (2015). This model considers the effects of kinetic instabilities and the rotational force on two plasma ion populations, expressing the energy change due to the decrease

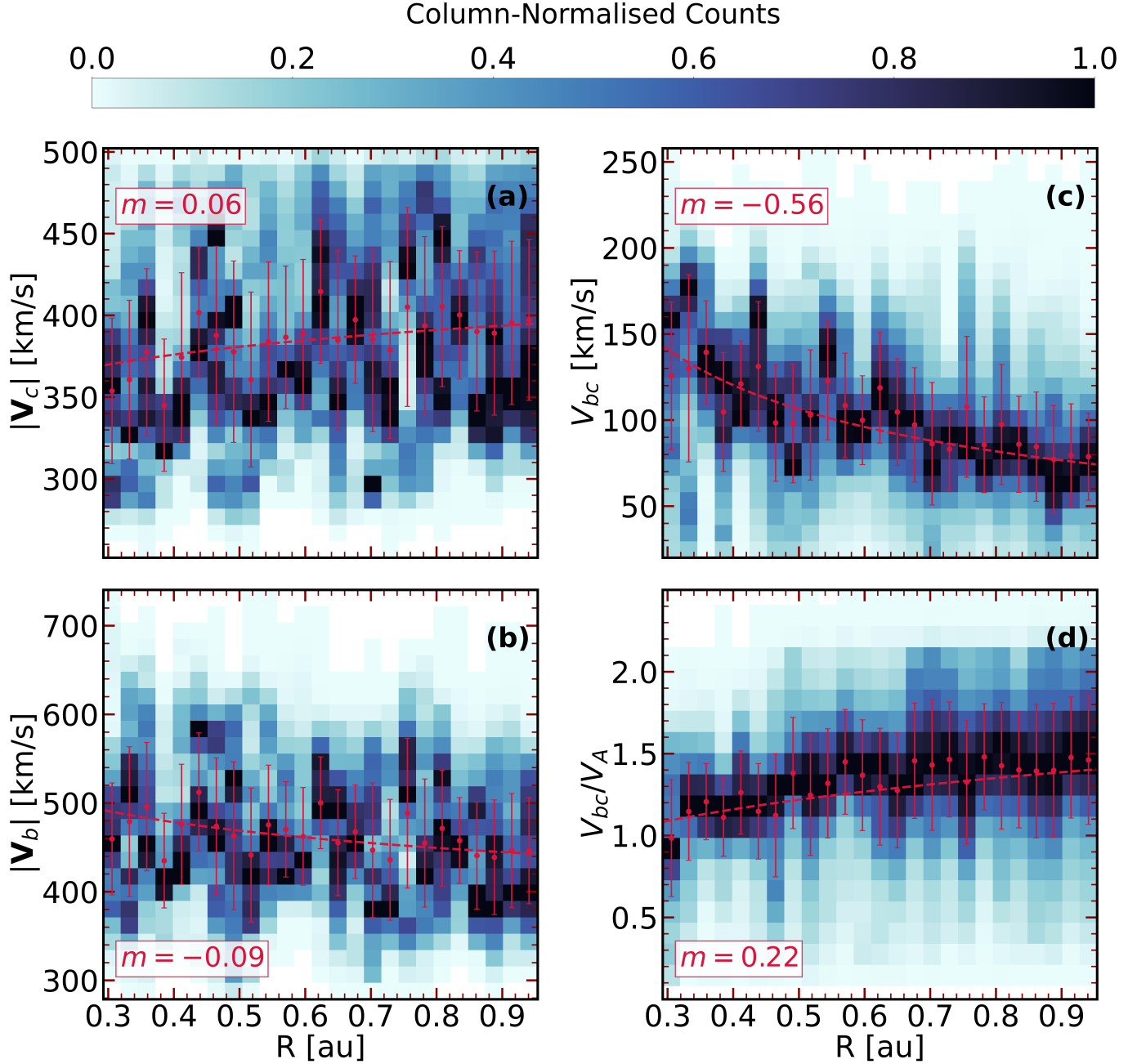


Figure 3. Radial profiles of the bulk velocity magnitude for the: (a) core and (b) beam protons. Panel (c) shows the radial profile of the core-beam drift velocity, V_{bc} , and panel (d) shows V_{bc} normalized to the local Alfvén speed, $V_A = |\mathbf{B}| / \sqrt{\mu_0 n_p m_p}$. The same format as Figure 2.

in the relative drift between them as

$$Q_{\text{flow}} = -\mu \rho_b \left[V_{b,r} \frac{\partial (V_{bc})^2}{\partial R} \frac{1}{2} + \frac{R(\Omega_\odot \sin \theta)^2 (V_{bc} \cos \psi_B)^2}{V_{c,r}} \right], \quad (9)$$

where ρ is the mass density, $\Omega_\odot$ is the Sun's angular velocity, θ is the colatitude in heliocentric spherical coordinates, ψ_B is the angle between $\mathbf{B}$ and the radial direction, $\hat{\mathbf{r}}$, and $\mu = \rho_c V_{c,r} / (\rho_c V_{c,r} + \rho_b V_{b,r})$.

To assess the significance of Q_{flow} in the overall proton thermal energy budget, we compare it to the empirical parallel,

$Q_{\parallel}$, and perpendicular, $Q_{\perp}$, heating rates, required to explain the observed core and beam temperature profiles, given by (G. F. Chew et al. 1956; B. D. G. Chandran et al. 2011; D. Verscharen et al. 2015)

$$Q_{\perp} = B n k_B V_r \frac{\partial}{\partial R} \left(\frac{T_{\perp}}{B} \right) \quad (10)$$

and

$$Q_{\parallel} = \frac{n^3 k_B V_r}{2B^2} \frac{\partial}{\partial R} \left(\frac{B^2 T_{\parallel}}{n^2} \right). \quad (11)$$

To calculate Q_{flow} , $Q_{\perp,c}$, $Q_{\parallel,c}$, $Q_{\perp,b}$, $Q_{\parallel,b}$, $Q_{\perp,p}$, and $Q_{\parallel,p}$, we use the fitted radial profiles of the relevant parameters from

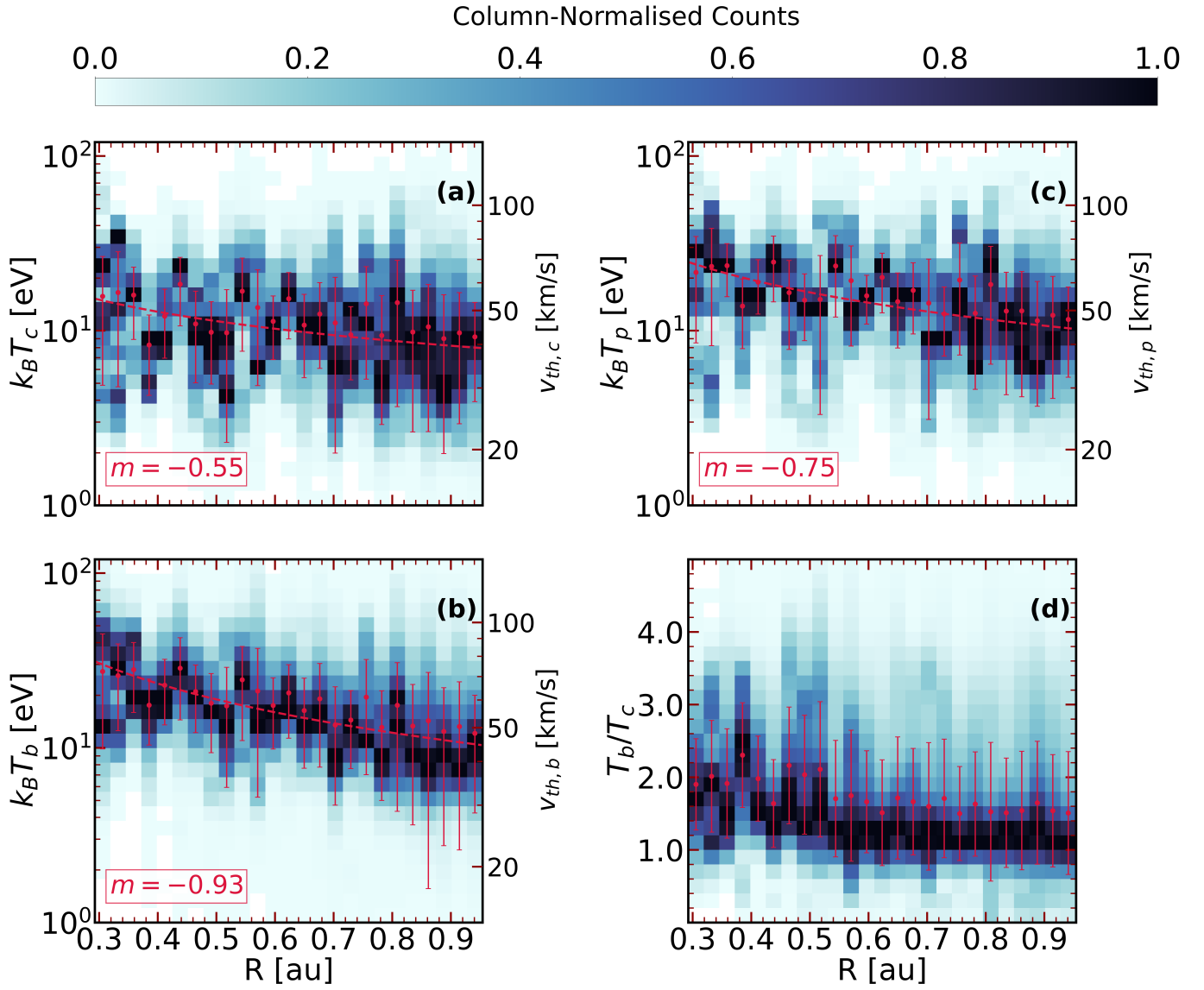


Figure 4. Radial profiles of the scalar temperature and thermal speed for the: (a) core, (b) beam, and (c) total protons. Panel (d) shows the radial profile of the temperature ratio, T_b/T_c . The same format as Figure 2.

Table 1. We calculate the radial derivative in Equation (9) analytically using the V_{bc} -fitted radial profile in Equation (8). We determine the magnetic field radial profile using the level 2 MAG data, yielding

$$|B| = 5.71 \text{ nT} \left(\frac{R}{1 \text{ au}} \right)^{-1.76}. \quad (12)$$

We set $\sin \theta = 1$, as Solar Orbiter remains near the ecliptic plane and $\Omega_\odot = 2.89 \times 10^{-6} \text{ s}$ (J. G. Beck 2000).

Figure 7 shows Q_{flow} , $Q_{\perp,c}$, $Q_{\parallel,c}$, $Q_{\perp,b}$, and $Q_{\parallel,b}$ against R . We plot the negative value of $Q_{\parallel,b}$ as it is negative across the entire radial range, while all other plotted heating rates are positive. Although not shown in Figure 7, $Q_{\parallel,p}$ is negative, while $Q_{\perp,p}$ is positive across the entire radial range, indicating that the protons, on average, experience parallel cooling and perpendicular heating. The core and beam parallel heating rates are weaker than the perpendicular heating rates, with $Q_{\perp,c}$ being the largest between them across the entire examined radial range. Q_{flow} is comparable to $Q_{\perp,c}$, exceeding

it at $R < 0.48 \text{ au}$, and decreasing to $Q_{\text{flow}}/Q_{\perp,c} \approx 1/3$ close to 1 au.

4. Discussion

The results presented above establish the radial evolution of the proton core and beam parameters and their associated heating rates. In the following section, we discuss these findings in the context of previous observations and current theoretical and numerical studies.

Overall, our results show that the radial evolution of the core and beam proton average parameters in the slow solar wind from Solar Orbiter observations is consistent with previous Helios observations. In both datasets, the density profiles follow spherically symmetric expansion with nearly constant radial velocity, while the bulk velocity and temperature profiles exhibit similar properties, with some minor differences in the slopes.

The observed decreases of $|V|_b$ and V_{bc} with R suggest that the beam protons are decelerated during expansion, whereas

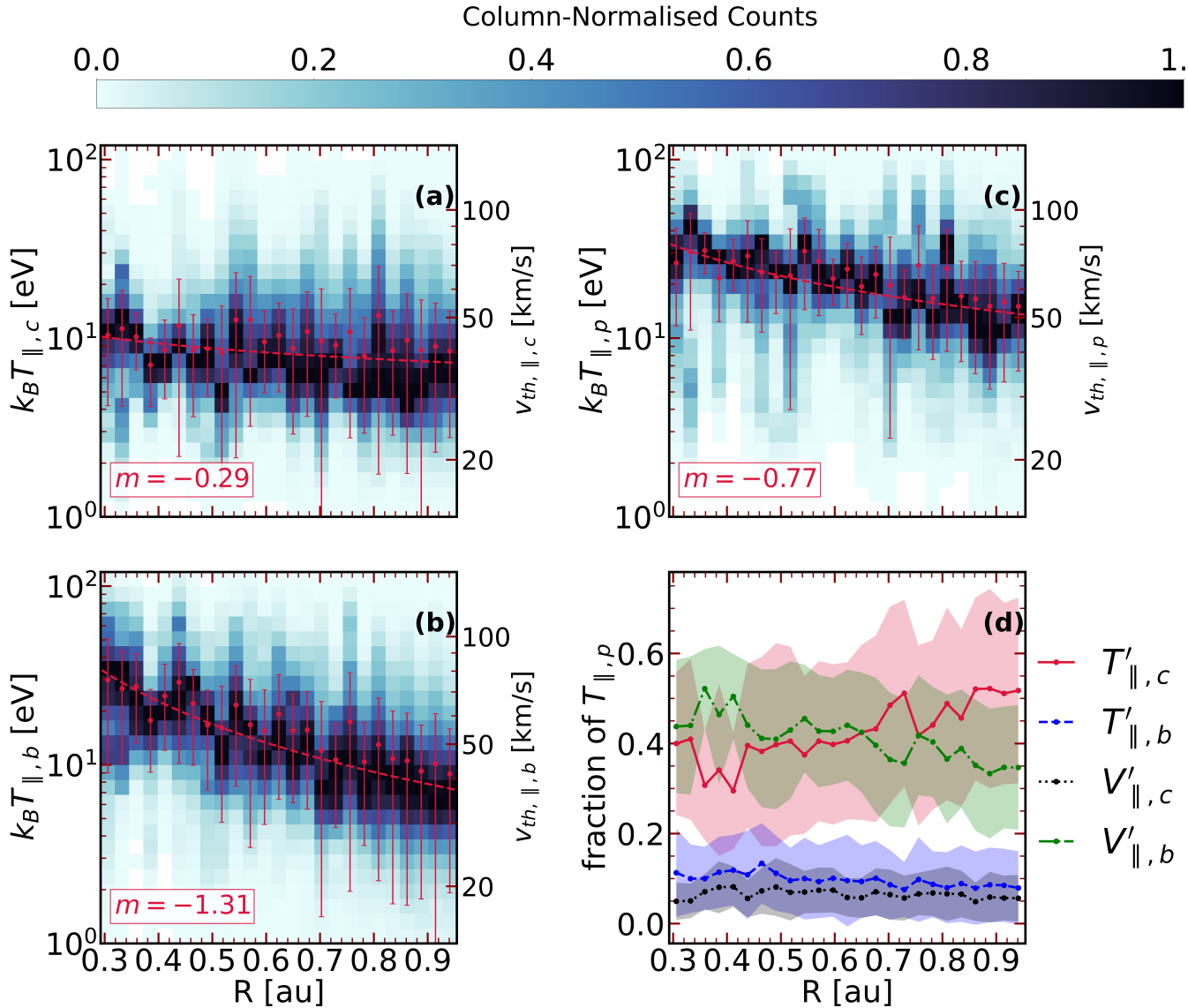


Figure 5. Radial profiles of the parallel temperature and thermal speed for the: (a) core, (b) beam, and (c) total protons. The same format as Figure 2. Panel (d) shows the fractional contributions to the second-order moment, $T'_{||,p}$, of the total proton population, arising from terms associated with the core and beam parallel temperatures, $T'_{||,c}$ and $T'_{||,b}$, and the core and beam parallel drifts, $V'_{||,c}$ and $V'_{||,b}$. The expressions for these quantities are given in Appendix A.6. The dots and shaded areas represent the mean and standard deviation of the fractional contributions for each bin.

the observed slight increase in $|V|_c$ with R suggests that some acceleration is occurring in the slow solar-wind core protons. The deceleration of the beam may result from resonant wave-particle interactions associated with beam-driven ion-scale instabilities, scattering the beam protons in velocity space (D. Verscharen et al. 2015; J. S. Bianco et al. 2026). Although the rotational force may also decelerate the beam, as the faster beam protons are forced to decelerate due to the spiral shape of the frozen-in interplanetary magnetic field, its contribution is expected to be minor over the examined radial range (J. F. McKenzie et al. 1979; D. Verscharen et al. 2015). Since the beam is decelerated, to conserve momentum, the core protons are accelerated while a portion of the momentum is transferred to waves (D. Verscharen et al. 2015). However, the observed acceleration of the core protons cannot be solely explained as momentum exchange between the core and the beam. Core proton acceleration may also partly reflect the conversion of thermal energy into bulk kinetic energy during

solar-wind expansion (B. Shivamoggi et al. 2021). An additional contribution may arise within stream interaction regions (SIRs), where slow solar wind is compressed and accelerated (I. G. Richardson 2018; T. Āurovcová et al. 2019).

The increase of V_{bc}/V_A with R indicates that the beam drift is not constrained by V_A . Hybrid expanding box simulations based on Helios fast solar-wind data (L. Matteini et al. 2012; J. S. Bianco et al. 2026) have also shown this trend, but with a steeper profile in V_{bc}/V_A compared to our results. This behavior contrasts with that of α -particles, which typically drift below V_A relative to the core protons in the slow solar wind (E. Marsch et al. 1982a; their Figure 11), leaving room for the interpretation that the beam and α -particle drifts arise from distinct mechanisms.

From Figure 5(d), we find that $T'_{||,c}$ and $V'_{||,b}$ are the dominant terms contributing to $T'_{||,p}$, while $T'_{||,b}$ and $V'_{||,c}$ are significantly smaller. This suggests that the parallel temperature of the beam protons is not a significant contributor to the total parallel

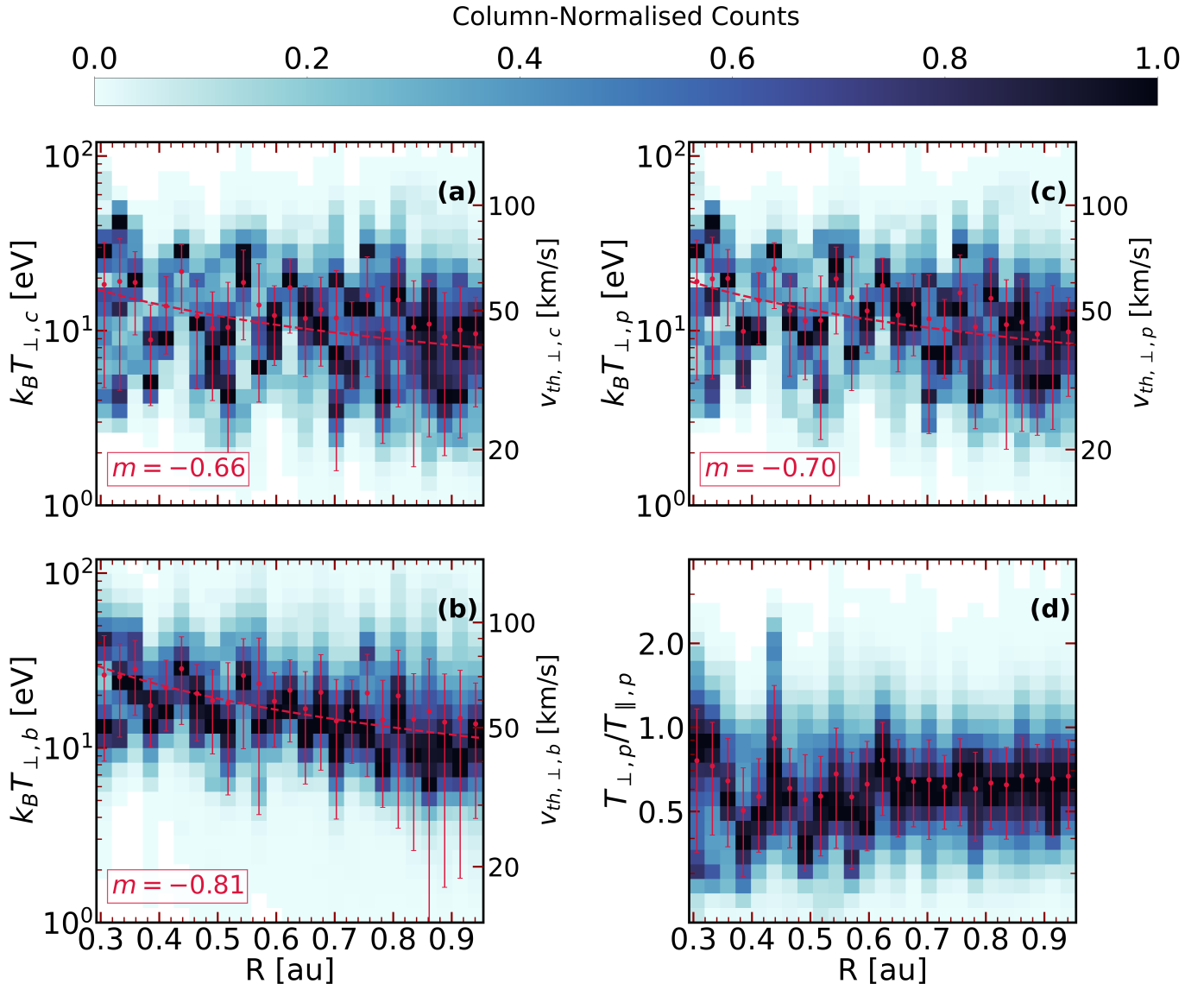


Figure 6. Radial profiles of the perpendicular temperature and thermal speed for the: (a) core, (b) beam, and (c) total protons. Panel (d) shows the radial profile of the temperature anisotropy, $T_{\perp,p}/T_{\parallel,p}$. The same format as Figure 2.

temperature, despite the beam being, on average, hotter than the core. While this is a consequence of the beam being significantly less dense than the core, we also find that the beam parallel drift term is comparable to the core parallel temperature term, indicating that the beam can play an important role in the overall proton thermal properties. Additionally, we observe a small decrease in the beam drift term and a small increase in the parallel core temperature term with R , which are likely a consequence of the beam deceleration.

Assuming a polytropic relationship between temperature and density, $T \propto n^{\gamma-1}$, where γ is the polytropic index, we can use the T_p and n_p radial profiles to investigate the thermodynamic evolution of the protons. This yields an effective index of $\gamma_p \approx 1.37$, which is smaller than the adiabatic prediction $\gamma = 5/3$ (assuming 3 degrees of freedom), indicating that the average total proton temperature decreases less than expected for adiabatic expansion.

According to the double adiabatic Chew, Goldberger, and Low (CGL) model, the parallel and perpendicular

temperatures are expected to evolve as $T_{\parallel,p} \propto n^2/B^2$ and $T_{\perp,p} \propto B$, respectively (G. F. Chew et al. 1956). For spherically symmetric expansion with constant radial velocity and a strictly radial magnetic field, the magnetic field strength scales as $|B| \propto 1/R^2$, and the total proton density scales as $n_p \propto 1/R^2$. Thus, under the double adiabatic model, the evolution of the parallel and perpendicular temperatures is $T_{\parallel,p} = \text{constant}$ and $T_{\perp,p} \propto R^{-2}$, leading to $T_{\perp,p}/T_{\parallel,p}$ decreasing with R . Eventually, the plasma becomes unstable to kinetic instabilities, which act to reduce the temperature anisotropy and restore stability to the plasma (L. Matteini et al. 2007).

Our observations, however, are inconsistent with the CGL predictions. We find that $T_{\perp,p}$ decreases less strongly than predicted by CGL, indicating perpendicular heating, while $T_{\parallel,p}$ decreases with R , indicating parallel cooling, in agreement with Helios slow wind observations (E. Marsch et al. 1983; P. Hellinger et al. 2013; T. Ďurovcová et al. 2019b). These heating trends are reflected in the total proton empirical heating rates, with $Q_{\perp,p}$ being positive and $Q_{\parallel,p}$ being negative.

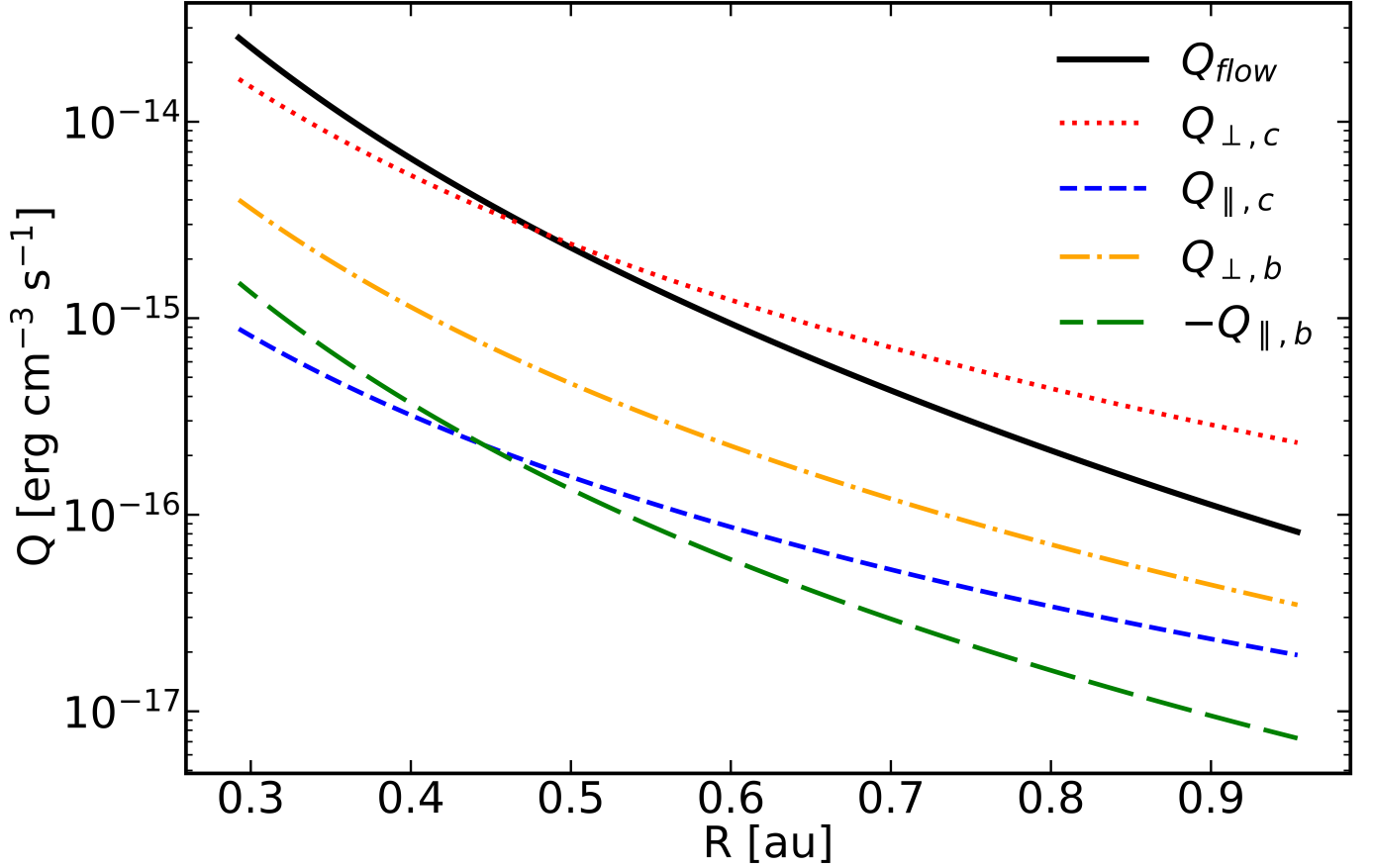


Figure 7. Radial profiles of Q_{flow} and the empirical heating rates $Q_{\perp,c}$, $Q_{\parallel,c}$, $Q_{\perp,b}$, and $Q_{\parallel,b}$. The value of $Q_{\parallel,b}$ is negative across the entire R range; thus, we plot its negative value.

Table 1

Core, Beam, and Total Proton Radial Profiles of Density, n , Radial Bulk Velocity, V_r , Bulk Velocity Magnitude, $|V|$, Scalar Temperature, T , Parallel Temperature, $T_{\parallel}$, and Perpendicular Temperature, $T_{\perp}$

Parameter	Core	Beam	Total
n	$5.43 \text{ cm}^{-3} \left(\frac{R}{1 \text{ au}}\right)^{-2.01}$	$0.59 \text{ cm}^{-3} \left(\frac{R}{1 \text{ au}}\right)^{-2.11}$	$6.24 \text{ cm}^{-3} \left(\frac{R}{1 \text{ au}}\right)^{-2.01}$
V_r	$395 \text{ km s}^{-1} \left(\frac{R}{1 \text{ au}}\right)^{0.06}$	$437 \text{ km s}^{-1} \left(\frac{R}{1 \text{ au}}\right)^{-0.09}$	$401 \text{ km s}^{-1} \left(\frac{R}{1 \text{ au}}\right)^{0.04}$
$ V $	$396 \text{ km s}^{-1} \left(\frac{R}{1 \text{ au}}\right)^{0.06}$	$440 \text{ km s}^{-1} \left(\frac{R}{1 \text{ au}}\right)^{-0.09}$	$405 \text{ km s}^{-1} \left(\frac{R}{1 \text{ au}}\right)^{0.04}$
T	$8.98 \times 10^4 \text{ K} \left(\frac{R}{1 \text{ au}}\right)^{-0.55}$	$1.15 \times 10^5 \text{ K} \left(\frac{R}{1 \text{ au}}\right)^{-0.93}$	$1.15 \times 10^5 \text{ K} \left(\frac{R}{1 \text{ au}}\right)^{-0.75}$
$T_{\parallel}$	$8.33 \times 10^4 \text{ K} \left(\frac{R}{1 \text{ au}}\right)^{-0.29}$	$7.89 \times 10^4 \text{ K} \left(\frac{R}{1 \text{ au}}\right)^{-1.31}$	$1.51 \times 10^5 \text{ K} \left(\frac{R}{1 \text{ au}}\right)^{-0.77}$
$T_{\perp}$	$8.96 \times 10^4 \text{ K} \left(\frac{R}{1 \text{ au}}\right)^{-0.66}$	$1.27 \times 10^5 \text{ K} \left(\frac{R}{1 \text{ au}}\right)^{-0.81}$	$9.40 \times 10^4 \text{ K} \left(\frac{R}{1 \text{ au}}\right)^{-0.70}$

Furthermore, we find that, on average, $T_{\perp,p}/T_{\parallel,p}$ does not decrease with R , but instead shows no clear radial dependence. $Q_{\text{flow}} \neq 0$ also indicates the inconsistency of our observations with the CGL model, as the decrease in the core-to-beam drift speed is a source of energy release that is not accounted for in the CGL model.

The violation of CGL suggests the presence of additional energy exchange processes, with kinetic instabilities driven by the proton temperature anisotropy likely playing a significant role. At small $T_{\perp,p}/T_{\parallel,p}$, the plasma is unstable to the firehose instability, which scatters particles from the leading edge of the beam population toward larger $u_{\perp}$ and smaller $u_{\parallel}$. This increases $T_{\perp,p}$ and decreases $T_{\parallel,p}$, resulting in parallel cooling

and perpendicular heating (D. Verscharen et al. 2015; J. S. Bianco et al. 2026). These theoretical and numerical results are consistent with the behavior we observe and suggest that firehose-driven scattering may contribute to the observed deviations from CGL evolution.

Regarding the evolution of the temperature anisotropy, observations of the slow solar wind exhibit significant variability across different streams, resulting in the absence of a clear radial dependence of the average $T_{\perp,p}/T_{\parallel,p}$, despite the existence of clearer correlations when observing individual streams (L. Matteini et al. 2007). This contrasts with fast wind observations, where a large scale negative correlation between $T_{\perp,p}/T_{\parallel,p}$ and R is observed between 0.3 and 1 au, until

$T_{\perp,p}/T_{\parallel,p}$ becomes sufficiently low, making the plasma become unstable to the firehose instability (E. Marsch et al. 1982b; L. Matteini et al. 2007).

Additionally, we find that Q_{flow} is of the same order of magnitude as the empirical proton heating rates, suggesting that the contribution of the deceleration of the core-to-beam drift in the overall proton energetics of the inner heliosphere slow solar wind is comparable to that of other local processes, such as plasma heating through the dissipation of turbulence, especially at smaller R .

5. Conclusions

We determine the radial evolution from the Sun of core and beam proton parameters in the slow solar wind from 0.3–1 au using Solar Orbiter data spanning a 2 yr period. To achieve this, we fit a double bi-Maxwellian model to the PAS proton observations using a novel PMLE technique.

We find that the core and beam radial profiles follow trends consistent with those observed previously by Helios in the slow solar wind (E. Marsch et al. 1982b; P. Hellinger et al. 2013; T. Ďurovcová et al. 2019b). Under the assumption of a nearly constant radial velocity, the core and beam densities follow the $1/R^2$ profile expected for spherically symmetric expansion, while the beam-to-core abundance ratio is approximately 10%. The core-to-beam drift speed decreases with radial distance, while the drift normalized to the local Alfvén speed slightly increases, spanning from 1 to 1.5. The proton temperature profiles do not follow the double adiabatic predictions (G. F. Chew et al. 1956), suggesting additional processes at work resulting in perpendicular heating and parallel cooling during expansion. The proton temperature anisotropy does not significantly vary with radial distance, with $T_{\perp,p}/T_{\parallel,p} \approx 0.65$ on average.

We determine the energy release due to the decrease in the core-to-beam drift speed, Q_{flow} , and compare it to the empirical core and beam heating rates required to explain the observed temperature profiles. Q_{flow} exceeds the beam heating rates and is comparable to $Q_{\perp,c}$ at $R < 0.48$ au, with $Q_{\text{flow}}/Q_{\perp,c}$ dropping to $\approx 1/3$ at 1 au. This suggests that the core-beam deceleration is energetically important in relation to other local proton heating processes in the inner heliosphere slow solar wind. A comprehensive analysis of the proton thermal energy budget evolution is required to better understand the mechanisms governing solar-wind proton energetics, which we leave for future work.

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under an open-source MIT License at https://github.com/Cioannou101/PMLE_SWA-PAS_fitting, with the specific version used for this work archived as PMLE-SWA-PAS-fitting v1.0.2 (doi:10.5281/zenodo.20595953). The dataset containing the core and beam proton parameters supporting this study (C. Ioannou 2026b) are publicly available on Zenodo at doi:10.5281/zenodo.20782283.

Appendix

A.1. K-means Clustering for Proton and α -particle Separation

We first integrate the VDF over Θ and Φ to obtain the 1D energy spectrum, such as the one shown in Figure A1. We then apply the K-means algorithm with the velocity, VDF, and the derivative of the VDF as inputs, using the scikit-learn Python library (F. Pedregosa et al. 2011), setting the number of clusters to 2. The algorithm separates the data into two clusters, the protons at lower energies (blue points in Figure A1), and the α -particles at higher energies. The velocity (energy) breakpoint between the two populations is the energy bin at the boundary between the two clusters, indicated by the blue-dashed line in Figure A1. We exclude all the data points (in 3D velocity space) with energies above this breakpoint from the fitting procedure.

A.2. Deviance Testing

The Poisson deviance, D , is defined as (P. McCullagh 2019)

$$D = 2(\ln \mathcal{L}_{\text{sat}} - \ln \mathcal{L}_{\text{fit}}), \quad (\text{A1})$$

where $\mathcal{L}_{\text{fit}}$ is the likelihood of the fitted model, and $\mathcal{L}_{\text{sat}}$ is the likelihood of the saturated model, which consists of as many free parameters as data points (i.e., $\lambda_i = C_i$ for all i). To account for the number of points, N , and number of free parameters in the model, p , we calculate the scaled or reduced deviance, (H. Heinzl & M. Mittlböck 2003)

$$D_{\text{red}} = \frac{2}{N - p}(\ln \mathcal{L}_{\text{sat}} - \ln \mathcal{L}_{\text{fit}}). \quad (\text{A2})$$

Larger values of D_{red} signify that the fitted model is less capable of matching the data, indicating a worse fit.

The deviance is not an absolute measure of goodness of fit; by itself it is not representative of the quality of the fit. Instead, it is useful as a comparative metric for evaluating the quality of similar fits. Thus, fits with comparatively high deviance values are likely to be unreliable. Figure A2 shows the distribution of D_{red} for all fits in our dataset. The distribution peaks at $D_{\text{red}} \approx 0.8$, associated with visually good fits across a range of plasma conditions. However, a large tail at higher values is evident, associated with fits that are visually poor or do not converge properly. Thus, we set a threshold of $D_{\text{red}} = 2$ and remove all fits with $D_{\text{red}} > 2$ from our analysis.

A.3. Overlap Metric

The overlap parameter, O , quantifies the overlap between the core and beam populations in velocity space and is defined as

$$O = \frac{\sum_i \min(f_{c,i}, f_{b,i})}{\sum_i f_{b,i}}, \quad (\text{A3})$$

where $f_{c,i}$ and $f_{b,i}$ are the core and beam VDF values, respectively, at velocity space pixel, i , where $f_b \neq 0$. O assumes values between 0 and 1, where small O values

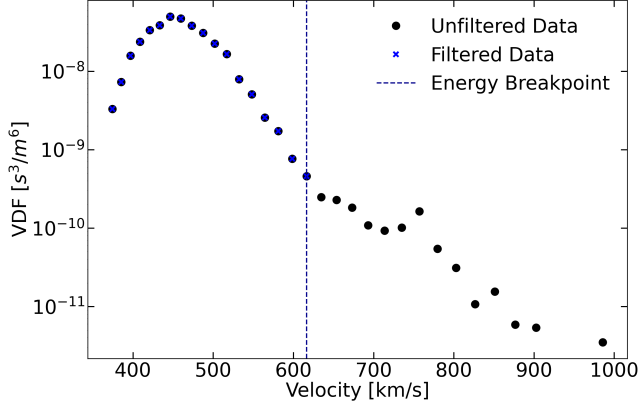


Figure A1. An example of K-means clustering applied to a PAS 1D energy spectrum.

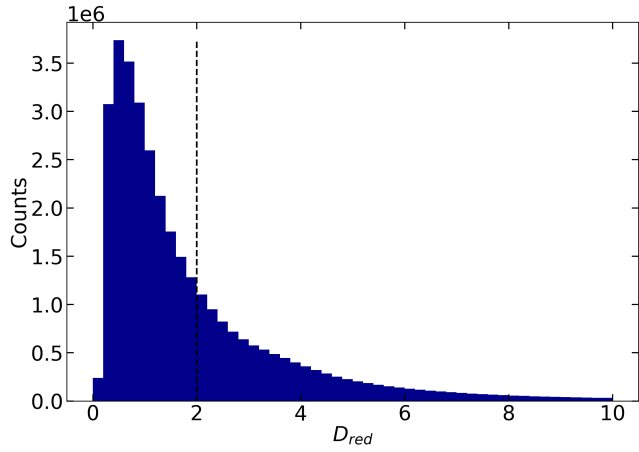


Figure A2. A histogram of the reduced Poisson deviance, D_{red} , for all fits in our dataset. The vertical dashed line indicates the threshold value of $D_{\text{red}} = 2$ above which we classify fits as poor and remove them from our analysis.

indicate that the core and beam populations are well separated in velocity space, while large O values indicate that the beam VDF is submerged under the core VDF. Figure A3 shows the distribution of O for all fits in our dataset. There is a large peak at $O \approx 1$, which is likely associated with fits with no clear beam population. Since we are only interested in fits with a clear and reliable beam component, we set a threshold of $O = 0.8$ and remove all fits with $O > 0.8$ from our analysis.

A.4. Derivation of Bulk Parameters from Partial Population Moments

A.4.1. Zeroth-order Moment

The zeroth moment of the VDF, f , yields the number density, n , and is defined as

$$n = \int f(\mathbf{v}) d^3v. \quad (\text{A4})$$

For multiple populations, s , we can write

$$f(\mathbf{v}) = \sum_s f_s(\mathbf{v}). \quad (\text{A5})$$

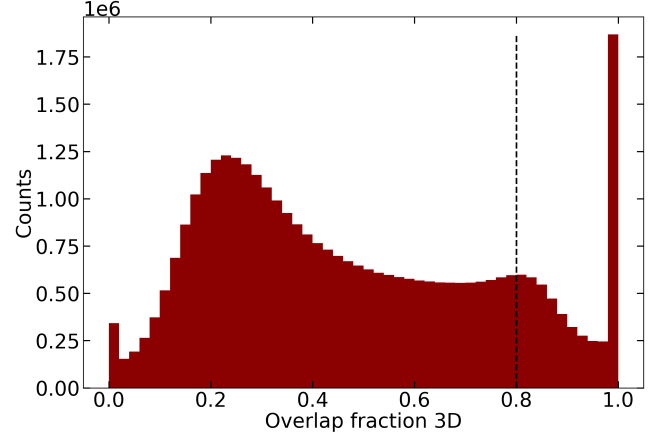


Figure A3. A histogram of the overlap parameter, O , for all fits in our dataset. The vertical dashed line indicates the threshold value of $O = 0.8$ above which we classify fits as having a significant overlap between core and beam populations.

Thus, the total number density is given by

$$\begin{aligned} n &= \int \sum_s f_s(\mathbf{v}) d^3v \\ n &= \sum_s \int f_s(\mathbf{v}) d^3v \\ n &= \sum_s n_s \end{aligned} \quad (\text{A6})$$

where n_s is the number density of population s .

A.4.2. First-order Moment

The first moment of f yields the bulk velocity, $\mathbf{V}$, and is defined as

$$\mathbf{V} = \frac{1}{n} \int \mathbf{v} f(\mathbf{v}) d^3v. \quad (\text{A7})$$

For multiple populations, the total bulk velocity is given by

$$\begin{aligned} \mathbf{V} &= \frac{1}{n} \int \mathbf{v} \sum_s f_s(\mathbf{v}) d^3v \\ \mathbf{V} &= \frac{1}{n} \sum_s \int \mathbf{v} f_s(\mathbf{v}) d^3v \\ \mathbf{V} &= \frac{1}{n} \sum_s n_s \mathbf{V}_s \\ \mathbf{V} &= \frac{\sum_s n_s \mathbf{V}_s}{\sum_s n_s} \end{aligned} \quad (\text{A8})$$

where $\mathbf{V}_s$ is the bulk velocity of population s .

A.4.3. Second-order Moment

The second moment of f yields the temperature tensor, $\mathbf{T}$, and is defined as

$$\mathbf{T} = \frac{m}{nk_B} \int (\mathbf{v} - \mathbf{V})(\mathbf{v} - \mathbf{V}) f(\mathbf{v}) d^3v \quad (\text{A9})$$

where m is the particle mass, and k_B is the Boltzmann constant. For multiple populations,

$$\mathbf{T} = \frac{m}{nk_B} \sum_s \int (\mathbf{v} - \mathbf{V})(\mathbf{v} - \mathbf{V}) f_s(\mathbf{v}) d^3v. \quad (\text{A10})$$

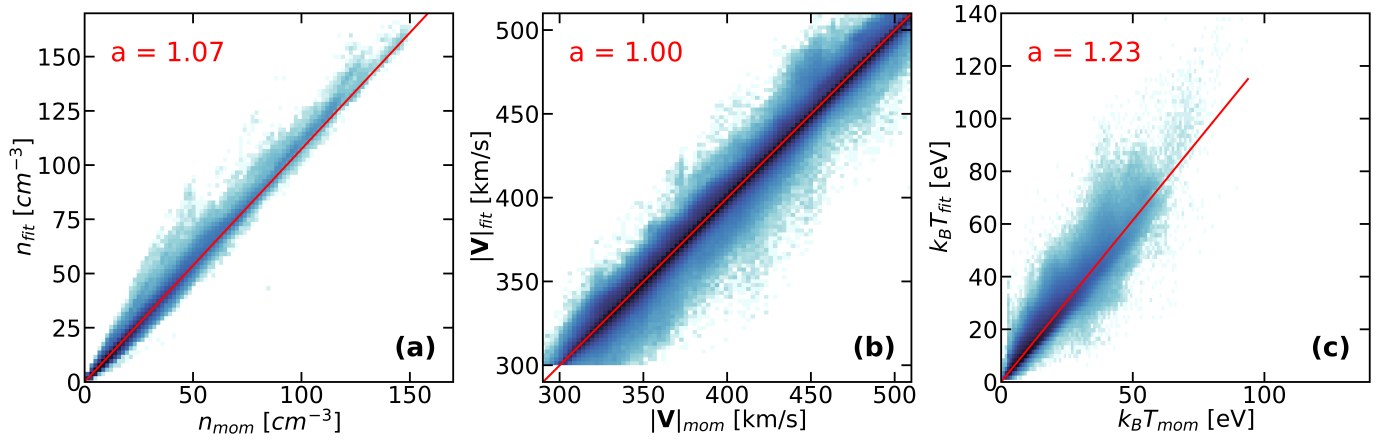


Figure A4. A 2D histogram comparing the fit and ground moment (a) density, (b) bulk speed, and (c) temperature. A linear regression between the fit and moment data is performed, indicated by the red curve, and the gradient, a , is shown in red at the top left corner of each panel.

For each population, we define

$$\delta \mathbf{v}_s \equiv \mathbf{v} - \mathbf{V}_s, \quad \Delta \mathbf{v}_s \equiv \mathbf{V}_s - \mathbf{V} \quad (\text{A11})$$

so that

$$\mathbf{v} - \mathbf{V} = \delta \mathbf{v}_s + \Delta \mathbf{v}_s. \quad (\text{A12})$$

Expand the dyadic product:

$$(\mathbf{v} - \mathbf{V})(\mathbf{v} - \mathbf{V}) = \delta \mathbf{v}_s \delta \mathbf{v}_s + \delta \mathbf{v}_s \Delta \mathbf{v}_s + \Delta \mathbf{v}_s \delta \mathbf{v}_s + \Delta \mathbf{v}_s \Delta \mathbf{v}_s. \quad (\text{A13})$$

Integrate term by term:

For the first term,

$$\int \delta \mathbf{v}_s \delta \mathbf{v}_s f_s(\mathbf{v}) d^3v = \frac{n_s k_B}{m} \mathbf{T}_s \quad (\text{A14})$$

following from Equation (A9). For the second term,

$$\int \delta \mathbf{v}_s \Delta \mathbf{v}_s f_s(\mathbf{v}) d^3v = \mathbf{0} \quad (\text{A15})$$

since

$$\int \delta \mathbf{v}_s f_s(\mathbf{v}) d^3v = \mathbf{0}. \quad (\text{A16})$$

Similarly, for the third term,

$$\int \Delta \mathbf{v}_s \delta \mathbf{v}_s f_s(\mathbf{v}) d^3v = \mathbf{0}. \quad (\text{A17})$$

Finally, for the fourth term,

$$\int \Delta \mathbf{v}_s \Delta \mathbf{v}_s f_s(\mathbf{v}) d^3v = n_s \Delta \mathbf{v}_s \Delta \mathbf{v}_s, \quad (\text{A18})$$

since $\Delta \mathbf{v}_s$ is not dependent on $\mathbf{v}$ and can be taken out of the integral.

Summing all the terms together yields

$$\mathbf{T} = \frac{m}{n k_B} \sum_s \left[\frac{n_s k_B}{m} \mathbf{T}_s + n_s \Delta \mathbf{v}_s \Delta \mathbf{v}_s \right]. \quad (\text{A19})$$

A.5. Comparison with Moments

Figure A4 shows a comparison between the level 2 ground moments and the respective fit values for the total proton density, bulk speed magnitude in the SRF, and scalar temperature. To determine the total proton density, n_{fit} , bulk speed, V_{fit} , and temperature tensor, $\mathbf{T}_{\text{fit}}$, of the fits, we use Equations (A6), (A8),

and (A19), assuming that the core and beam are the only proton populations present. This determination leads to the following expressions for the total proton fit density, bulk velocity, and temperature tensor:

$$n_{\text{fit}} = n_c + n_b, \quad (\text{A20})$$

$$\mathbf{V}_{\text{fit}} = \frac{n_c \mathbf{V}_c + n_b \mathbf{V}_b}{n_c + n_b}, \quad (\text{A21})$$

and

$$\begin{aligned} \mathbf{T}_{\text{fit}} = & \frac{n_c \mathbf{T}_c + n_b \mathbf{T}_b}{n_c + n_b} \\ & + \frac{m_p}{k_B(n_c + n_b)} [n_c (\mathbf{V}_{\text{fit}} - \mathbf{V}_c)^2 \\ & + n_b (\mathbf{V}_{\text{fit}} - \mathbf{V}_b)^2] \hat{\mathbf{V}}_{bc} \hat{\mathbf{V}}_{bc}, \end{aligned} \quad (\text{A22})$$

where $\hat{\mathbf{V}}_{bc}$ is the unit vector of the core-to-beam drift velocity. The scalar temperature, T_{fit} , is then determined by taking the trace of $\mathbf{T}_{\text{fit}}$.

There is strong agreement between the fit and moment bulk velocity magnitude. The fitted density and temperature are on average larger than the moments. This is not surprising as the moments are computed on a discrete velocity grid after removing higher-energy VDF data, where α -particles reside, meaning that the calculation does not take into account data in the parameter space where protons and α -particles overlap. This problem is alleviated by fitting the data, as the model accounts for the parameter space beyond the observations, resulting in systematically higher values than the moments. The effect is more pronounced when determining the temperature, as the outer parts of the VDFs make stronger contributions to the temperature than to lower moments, which is why the discrepancy between the fits and the moment temperatures is larger when compared to the densities.

A.6. Contributions to the Parallel Second-order Moment

Since the core-to-beam drift in our model is strictly parallel to the local magnetic field, the drift of each population relative to the total proton bulk velocity is also strictly in the parallel direction. Consequently, the core and beam parallel drift terms in Equation (A22) are nonzero and thus contribute to the parallel second-order moment of the total proton population, whereas the respective terms in the perpendicular direction are

zero. Following Equation (A22), the total proton parallel temperature, $T_{\parallel,p}$, is given by

$$T_{\parallel,p} = \frac{n_c T_{\parallel,c} + n_b T_{\parallel,b}}{n_c + n_b} + \frac{m_p}{k_B(n_c + n_b)} \times [n_c(V_{\parallel,p} - V_{\parallel,c})^2 + n_b(V_{\parallel,p} - V_{\parallel,b})^2], \quad (\text{A23})$$

where n_c and n_b are the core and beam densities, $T_{\parallel,c}$ and $T_{\parallel,b}$ are the core and beam parallel temperatures, $V_{\parallel,c}$ and $V_{\parallel,b}$ are the core and beam parallel bulk speeds, and $V_{\parallel,p}$ is the total proton parallel bulk speed.

We can decompose Equation (A23) into the population temperature and drift terms:

$$T_{\parallel,p} = T'_{\parallel,c} + T'_{\parallel,b} + V'_{\parallel,c} + V'_{\parallel,b} \quad (\text{A24})$$

where

$$T'_{\parallel,c} = \frac{n_c T_{\parallel,c}}{n_c + n_b} \quad (\text{A25})$$

is the parallel second-order moment term associated with the core parallel temperature,

$$T'_{\parallel,b} = \frac{n_b T_{\parallel,b}}{n_c + n_b} \quad (\text{A26})$$

is the parallel second-order moment term associated with the beam parallel temperature,

$$V'_{\parallel,c} = \frac{m_p n_c (V_{\parallel,\text{fit}} - V_{\parallel,c})^2}{k_B(n_c + n_b)} \quad (\text{A27})$$

is the parallel second-order moment term associated with the core parallel drift, and

$$V'_{\parallel,b} = \frac{m_p n_b (V_{\parallel,\text{fit}} - V_{\parallel,b})^2}{k_B(n_c + n_b)} \quad (\text{A28})$$

is the parallel second-order moment term associated with the beam parallel drift.

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